Verona Public School District
Curriculum Overview

Pre-Calculus CP/H

Curriculum Committee Members:
Danielle Mutovic
Michelle DellaFortuna

Supervisor:
Glen Stevenson

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Verona Public Schools
121 Fairview Ave., Verona, NJ 07044
www.veronaschools.org

Verona Public Schools Mission Statement:

The mission of the Verona Public Schools, the center of an engaged and supportive community, is to empower students to achieve their potential as active learners and productive citizens through rigorous curricula and meaningful, enriching experiences.

Course Description:

This course is designed to enhance students' preparation for Honors and AP Calculus. The course will focus on improving students' knowledge of trigonometric and other types of functions, including polynomial, rational, exponential and logarithmic functions. Other topics include: Analytic Trigonometry, Applications of Trigonometry, Sequences and Series, Conic Sections, Parametric Equations, Polar Coordinates, Limits. Much of this course involves real-world applications and mathematical modeling. Student must provide their own TI NSpire CAS graphing calculator; this will be an important tool that will routinely be used in instruction.

Prerequisite(s):

Algebra II

Verona High School — PreCalculus

Standard 8: Technology Standards

8.1: Educational Technology: *All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge.*	**8.2: Technology Education, Engineering, Design, and Computational Thinking - Programming:** *All students will develop an understanding of the nature and impact of technology, engineering, technological design, computational thinking and the designed world as they relate to the individual, global society, and the environment.*
X A. Technology Operations and Concepts **X** B. Creativity and Innovation C. Communication and Collaboration D. Digital Citizenship E. Research and Information Fluency **X** F. Critical thinking, problem solving, and decision making	**X** A. The Nature of Technology: Creativity and Innovation B. Technology and Society C. Design D. Abilities for a Technological World E. Computational Thinking: Programming

SEL Competencies and Career Ready Practices

Social and Emotional Learning Core Competencies: *These competencies are identified as five interrelated sets of cognitive, affective, and behavioral capabilities*	**Career Ready Practices:** *These practices outline the skills that all individuals need to have to truly be adaptable, reflective, and proactive in life and careers. These are researched practices that are essential to career readiness.*
Self-awareness: The ability to accurately recognize one's emotions and thoughts and their influence on behavior. This includes accurately assessing one's strengths and limitations and possessing a well-grounded sense of confidence and optimism.	**X** CRP2. Apply appropriate academic and technical skills. CRP9. Model integrity, ethical leadership, and effective management. **X** CRP10. Plan education and career paths aligned to personal goals.
Self-management: The ability to regulate one's emotions, thoughts, and behaviors effectively in different situations. This includes managing stress, controlling impulses, motivating oneself, and setting and working toward achieving personal and academic goals.	CRP3. Attend to personal health and financial well-being. **X** CRP6. Demonstrate creativity and innovation. **X** CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP11. Use technology to enhance productivity.
Social awareness: The ability to take the perspective of and empathize with others from diverse backgrounds and cultures, to understand social and ethical norms for behavior, and to recognize family, school, and community resources and supports.	**X** CRP1. Act as a responsible and contributing citizen and employee. CRP9. Model integrity, ethical leadership, and effective management.
Relationship skills: The ability to establish and maintain healthy and rewarding relationships with diverse individuals and groups. This includes communicating clearly, listening actively, cooperating, resisting inappropriate social pressure, negotiating conflict constructively, and seeking and offering help when needed.	**X** CRP4. Communicate clearly and effectively and with reason. CRP9. Model integrity, ethical leadership, and effective management. CRP12. Work productively in teams while using cultural global competence.
Responsible decision making: The ability to make constructive and respectful choices about personal behavior and social interactions based on consideration of ethical standards, safety concerns, social norms, the realistic evaluation of consequences of various actions, and the well-being of self and others.	**X** CRP5. Consider the environmental, social, and economic impact of decisions. CRP7. Employ valid and reliable research strategies. **X** CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP9. Model integrity, ethical leadership, and effective management.

Standard 9: 21st Century Life and Careers

9.1: Personal Financial Literacy: *This standard outlines the important fiscal knowledge, habits, and skills that must be mastered in order for students to make informed decisions about personal finance. Financial literacy is an integral component of a student's college and career readiness, enabling students to achieve fulfilling, financially-secure, and successful careers.*	**9.2: Career Awareness, Exploration & Preparation:** *This standard outlines the importance of being knowledgeable about one's interests and talents, and being well informed about postsecondary and career options, career planning, and career requirements.*	**9.3: Career and Technical Education:** *This standard outlines what students should know and be able to do upon completion of a CTE Program of Study.*
X A. Income and Careers B. Money Management C. Credit and Debt Management **X** D. Planning, Saving, and Investing **X** E. Becoming a Critical Consumer F. Civic Financial Responsibility G. Insuring and Protecting	A. Career Awareness (K-4) B. Career Exploration (5-8) **X** C. Career Preparation (9-12)	A. Agriculture, Food & Natural Res. B. Architecture & Construction C. Arts, A/V Technology & Comm. D. Business Management & Admin. E. Education & Training F. Finance G. Government & Public Admin. H. Health Science I. Hospital & Tourism J. Human Services K. Information Technology L. Law, Public, Safety, Corrections & Security M. Manufacturing N. Marketing **X** O. Science, Technology, Engineering & Math P. Transportation, Distribution & Log.

Course Materials

Core Instructional Materials: *These are the board adopted and approved materials to support the curriculum, instruction, and assessment of this course.*	**Differentiated Resources**: *These are teacher and department found materials, and also approved support materials that facilitate differentiation of curriculum, instruction, and assessment of this course.*
• TI-Nspire CAS Calculator • Pre-Calculus with Limits, a Graphical Approach	• Khan Academy • Delta Math • Kuta Software • TI-Nspire CX CAS Student Software • SmartBoard and SMART Notebook 10 Software • Google Classroom, Sheets, Slides, and Documents

Verona High School — PreCalculus

Unit 1: Algebra Review	Unit Duration: 4 Weeks

Stage 1: Desired Results

Established Goals:

A-APR4. Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.*

A-SSE2. Use the structure of an expression to identify ways to rewrite it. *For example, see* $x4 – y4$ *as* $(x^2)^2 – (y^2)^2$, *thus recognizing it as a difference of squares that can be factored as* $(x^2 – y^2)(x^2 + y^2)$.

Transfer Goal:

Students will be able to independently use their learning to…

- think metacognitively about algebraic operations and manipulation.

Students will understand that:

U1. Solving algebraic equations is a basic test in their knowledge of the order of operations
U2. It can be determined if an expression is (or is not) completely simplified
U3. Basic rules of equality are the key to ensure accuracy in mathematics

Essential Questions:

Q1. How do we solve an unfamiliar equation?
Q2. How do we know if an expression is completely simplified?
Q3. Besides checking our work, what are some strategies to help us gain confidence that we are solving a problem correctly?

Students will know:

K1. The algorithm for converting between exponents and radicals
K2. The various types of factorable expressions
K3. The algorithm for factoring the difference of perfect cubes
K4. Algorithms for solving various equations and inequalities

Students will be able to:

A1. Change back and forth between exponents and radicals
A2. Factor
A3. Solve various types of algebraic equations and inequalities
A4. Point out common algebraic errors

Stage 2: Acceptable Evidence

Performance Task & Unit Assessments:

Assessments -

- Quiz #1
- Quiz #2
- Quiz #3
- Unit Test

Transfer Task: Error Analysis
An activity will ask students to correct common algebraic errors. They will correct these errors, and explain the student's thinking.

Other Evidence:
Students will show that they have achieved Stage 1 goals by:

Formal:
- Providing written/oral response to the EQs
- Passing all quizzes on basic concepts in unit.

Informal:
- Identify parts of a polynomial while working towards other parts of problems
- Students should also troubleshoot examples and explain misapplications of the conventions or properties.

Reference Materials

- Pre-Calculus with Limits, a Graphical Approach
- TI-Nspire Lesson Activities: https://education.ti.com/en/timathnspired/us/precalculus
- Guided Notes #1 - Day 1
- Guided Notes #1 - Day 2
- Guided Notes #2
- Guided Notes #3
- Guided Notes #4
- Khan Academy Links

Verona High School

PreCalculus

Unit 2: Functions	Unit Duration: 4 Weeks

Stage 1: Desired Results

Established Goals:

F-IF4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

F-IF7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

F-BF1. Write a function that describes a relationship between two quantities.

c. (+) Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.

F-BF3. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

Include recognizing even and odd functions from their graphs and algebraic expressions for them.

F-BF4. Find inverse functions.

a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, f(x) =2x+3 or f(x) = (x+1)/(x–1) for $x \neq 1$.

b. (+) Verify by composition that one function is the inverse of another.

c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.

d. (+) Produce an invertible function from a non-invertible function by restricting the domain.

Transfer Goal:

Students will be able to <u>independently</u> use their learning to…

- analyze the graph of a real-life function and use their insights to help them make informed decisions about that function.

Students will understand that:

U1. Transformations of functions can be represented graphically or algebraically

U2. Graphical representations of functions can provide additional insight into a function's behavior

Essential Questions:

Q1. Can all transformations of basic functions be represented in a predictable algebraic way?

Q2. When is it more appropriate to analyze a function algebraically? Graphically?

Students will know:

K1. Graphs of basic functions

K2. The relationship between a function and its inverse

Students will be able to:

A1. Evaluate functions for any given input

A2. Write equations for graphs

A3. Graph common functions and transformations without a calculator

A4. Compose functions

A5. Find function inverses

Stage 2: Acceptable Evidence

Performance Task & Unit Assessments:

Assessments -

- Quiz #1
- Quiz #2
- Unit Test

Transfer Task: Function Story

Students will complete one of the following tasks: a function dance, or a function story. In either case, students will utilize the characteristics of functions to either give meaning to a story, or to provide beauty in a dance.

Other Evidence:

Students will show that they have achieved Stage 1 goals by:

Formal:

- Providing written/oral response to the EQs
- Passing all quizzes on basic concepts in unit.

Informal:

- Identify the inverse of a function rule
- Analyze the graphs of various functions
- Create new function rules
- Students should also troubleshoot examples and explain misapplications of the conventions or properties.

Reference Materials

- Pre-Calculus with Limits, a Graphical Approach
- TI-Nspire Lesson Activities: https://education.ti.com/en/timathnspired/us/precalculus
- Guided Notes #1
- Guided Notes #2
- Guided Notes #3
- Guided Notes #4
- Guided Notes #5
- Guided Notes #6
- Guided Notes #7
- Family of Functions Handout

Verona High School — PreCalculus

Unit 3: Fundamental Theorem of Algebra and Rational Functions	Unit Duration: 6 Weeks

Stage 1: Desired Results

Established Goals:

N-CN4. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.

N-CN5. (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $(-1 + \sqrt{3}\,i)^3 = 8$ *because* $(-1 + \sqrt{3}\,i)$ *has modulus* 2 *and argument* 120°.

N-CN6. (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

N-VM1. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., v, $|v|$, $||v||$, v).

N-VM4. (+) Add and subtract vectors.

a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.

b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.

c. Understand vector subtraction $v - w$ as $v + (-w)$, where $-w$ is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.

N-CN7. Solve quadratic equations with real coefficients that have complex solutions.

N-CN8. (+) Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x - 2i)$.

N-CN9. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

A-APR2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.

A-APR3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

A-APR6. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

F-IF7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

Transfer Goal:

Students will be able to <u>independently</u> use their learning to…

- analyze the graph of a polynomial and rational function.

Students will understand that:

U1. Polynomial and rational functions can be sketched quickly and accurately using nothing but algebraic skills

U2. All zeroes of a polynomial function can be found by using division to break the function down into the product of linear and quadratic factors

Essential Questions:

Q1. What makes an accurate sketch of a polynomial function?

Q2. What makes a good window for the graph of a polynomial function?

Q3. What is the best way to find the zeroes of a polynomial function?

Students will know:

K1. Fundamental theorem of algebra

K2. Algorithms to find asymptotes

K3. Rational zero test

Students will be able to:

A1. Divide polynomials using long and synthetic division

A2. Find max/min of quadratics

A3. Sketch polynomial and rational functions

A4. Find all zeroes of polynomial functions

A5. Perform operations with complex numbers

A6. Write a polynomial function given its zeroes

Stage 2: Acceptable Evidence

Performance Task & Unit Assessments:

Assessments -

- Lesson 1 Exit Ticket
- Quiz #1
- Quiz #2
- Quiz #3
- Birthday Polynomial Project
- Unit Test

Transfer Task: Angry Bird Project

Students will find the equation of the flight of an angry bird. Students will analyze the properties of this path, and they will discuss how these properties affect the flight of the bird.

Other Evidence:

Students will show that they have achieved Stage 1 goals by:

Formal:

- Providing written/oral response to the EQs
- Passing all quizzes on basic concepts in unit.

Informal:

- Identifying all aspects of a rational function's graph, comparing and contrasting polynomial functions
- Troubleshoot examples explaining errors in reasoning and misuse of conventions or properties
- Being able to explain why concepts regarding fractions are essential to master BEFORE fully understanding Rational expressions

Reference Materials

- Pre-Calculus with Limits, a Graphical Approach
- TI-Nspire Lesson Activities: https://education.ti.com/en/timathnspired/us/precalculus
- Guided Notes #1
- Guided Notes #2
- Guided Notes #3
- Guided Notes #4
- Guided Notes #5
- Guided Notes #6 Day 1
- Guided Notes #6 Day 2
- Guided Notes #7
- Guided Notes #8

Verona High School — PreCalculus

Unit 4: Exponential and Logarithmic Functions | Unit Duration: 5 Weeks

Stage 1: Desired Results

Established Goals:

F-IF7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

F-IF8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay.*

F-BF5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

L-FE4. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

Transfer Goal:

Students will be able to <u>independently</u> use their learning to…

- investigate important real-world phenomena modeled by exponential and logarithmic functions from a graphical and an algebraic perspective.

Students will understand that:

U1. Logarithmic functions are the inverse of exponential functions
U2. Exponential and logarithmic functions model real-world phenomena
U3. Inverse functions allow us to solve equations algebraically

Essential Questions:

Q1. Why do we need the logarithm function?
Q2. What real-world phenomena are modeled by exponential or logarithmic functions?

Students will know:

K1. The properties of logs
K2. The relationship between exponential and log functions
K3. Different real-life applications of exponential and log functions

Students will be able to:

A1. Graph exponential and log functions
A2. Evaluate exponential and log functions
A3. Manipulate log expressions
A4. Solve log and exponential equations
A5. Apply compounded interest formulas

Stage 2: Acceptable Evidence

Performance Task & Unit Assessments:

Assessments -

- Quiz #1
- Quiz #2
- Unit Test
- Population Analysis Project

Transfer Task:

Savings Account Project: Students will compare the population growth of two demographic groups. They will make predictions, and analyze how these changes make affect them individually, as a community, and as a country.

Other Evidence:

Students will show that they have achieved Stage 1 goals by:

Formal:
- Providing written/oral response to the EQs
- Passing all quizzes on basic concepts in unit.

Informal:
- Students should also troubleshoot examples and explain misapplications of the conventions or properties
- Students can explain how to use logarithms
- Students can explain what exponential growth/decay is and how it differs from other types of growth/decay
- Students can explain the properties of inverse functions as related to exponential and logarithmic functions

Reference Materials

- Pre-Calculus with Limits, a Graphical Approach
- TI-Nspire Lesson Activities: https://education.ti.com/en/timathnspired/us/precalculus
- Khan Academy Links
- Guided Notes #1
- Guided Notes #2
- Guided Notes #3
- Guided Notes #4
- Guided Notes #5
- Guided Notes #6
- Guided Notes #7
- Guided Notes #8

Verona High School

PreCalculus

Unit 5: Triangle, Right Triangle and Unit Circle Trigonometry	Unit Duration: 6 Weeks

Stage 1: Desired Results

Established Goals:

F.TF.1 - Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
F.TF.2 - Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
F.TF.3 - Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.
F.TF.4 - Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
F.TF.5 - Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
F.TF.6 - Understand that restricting a trigonometric function to a domain on which it is always increasing or decreasing allows its inverse to be constructed.
F.TF.7 - Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate that solutions using technology and interpret them in terms of the context.
F.TF.8 - Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.
F.TF.9 - Prove the addition and subtraction formulas for sine, cosine and tangent and use them to solve problems.
G.SRT.6 - Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
G.SRT.7 - Explain and use the relationship between the sine and cosine of complementary angles.
G.SRT.8 - Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
G.SRT.9 - Derive the formula $A = 1/2\ ab\sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
G.SRT.10 - Prove the Laws of Sines and Cosines and use them to solve problems.
G.SRT.11 - Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Transfer Goal:

Students will be able to <u>independently</u> use their learning to…

- understand the implications of right triangle trigonometry in the unit circle and non-right triangle trigonometry.

Students will understand that:

U1. Trig functions are ratios of sides of right triangles
U2. Trig functions evaluated at a certain angle are constant, regardless of the size of the triangle
U3. Trig identities allow us to express trig expressions in different, but equivalent ways
U4. The Unit Circle allows us to evaluate trig functions at quadrantal angles
U5. Reference angles allow us to evaluate trig functions at angles greater than 90 degrees
U6. Radians are often a more appropriate way of describing angles
U7. Tangent and Secant are named after their relationship to the unit circle
U8. The LoC, LoS and Area formula are generalized versions of Pythagorean Theorem, SohCahToa and A=1/2bh
U9. Trigonometry allows you to find distances or angles that you could not have measured

Essential Questions:

Q1. What does evaluating a trig function at a given angle mean in real life?
Q2. How does changing the size of a right triangle affect the sine, cosine and tangent of its angles?
Q3. What is the value in knowing trig identities?
Q4. How does the unit circle solidify (and enhance) our understanding of trig functions?
Q5. What is the purpose of measuring angles in radians?
Q6. What is the relationship of the Law of Cosines, Law of Sines and area formula to theorems you previously learned?
Q7. What is the utility of the Law of Sines and the Law of Cosines?

Students will know:

K1. Basic trig identities
K2. Pythagorean identities
K3. Cofunction relationships
K4. Basic trig functions of 30, 45 60 degree angles
K5. The relationship between (cos, sin) and (x, y) on the unit circle
K6. Sin, cos and tan of 30, 45, 60
K7. Sin, cos and tan of quadrantal angles
K8. Law of Cosines
K9. Law of Sines
K10. Area Formula

Students will be able to:

A1. Verify identities
A2. Simplify trig expressions
A3. Solve basic right triangle trig word problems
A4. Convert between radians and degrees
A5. Evaluate a trigonometric function for any multiple of 30, 45, 60 or 90
A6. Find all six trig functions of an angle, given one
A7. Fill out a unit circle (angles in radians, coordinates of points)
A8. Solve area problems
A9. Solve force problems

Stage 2: Acceptable Evidence

Performance Task & Unit Assessments:

Assessments -

- Exact Values Quiz
- Right Triangle Trig Quiz - CP
- Identities Quiz
- Law of Cosines Quiz - CP
- Area and The Law of Sines Quiz - CP
- General Angles Quiz - CP
- Radian Measure and the Unit Circle Quiz - CP
- Right Triangle & Unit Circle Trig Exam Version 1 - Honors
- Right Triangle & Unit Circle Trig Exam Version 2 - Honors
- Right Triangle & Unit Circle Trig Exam Version 3 - Honors
- Right Triangle & Unit Circle Trig Exam Version 4 - Honors
- Right Triangle & Unit Circle Trig Exam Version 5 - Honors
- Right Triangle & Unit Circle Trig Exam - Version 1 - CP
- Right Triangle & Unit Circle Trig Exam - Version 2 - CP
- Right Triangle & Unit Circle Trig Exam - Version 3 - CP

Transfer Task: Angle of Elevation of the Sun

Students will be asked to take measurements of their shadow at two different times of day (at least two hours apart). In order to calculate the angle of elevation of the sun, students must also have an accurate reading of their heights and shadows. Student will construct two triangles from theses measurements and calculate the angle of elevation using inverse trig. Students will then be asked to reflect on their findings and answer follow up questions.

Other Evidence:
Students will show that they have achieved Stage 1 goals by:

Formal:
- Providing written/oral response to the EQs
- Passing all quizzes on basic concepts in unit.

Informal:
- Recreating the Unit Circle
- Generating exact values of certain trig functions without a calculator
- Appropriately applying Right-Triangle and Non-Right Triangle Formulas
- Students should also troubleshoot examples and explain misapplications of the conventions or properties.

Reference Materials

- Pre-Calculus with Limits, a Graphical Approach
- TI-Nspire Lesson Activities: https://education.ti.com/en/timathnspired/us/precalculus
- Honors - Right Triangle & Unit Circle Trig Teacher Notes
- Honors - Right Triangle & Unit Circle Trig Student Notes
- Blank Unit Circle
- Trig Squares Game
- Intro To Radian Measure Activity
- Trig Identities Formula Sheet
- Honors - Non-Right Triangle Trig - Teacher Notes
- Honors - Non-Right Triangle Trig - Student Notes
- Unit Circle Activity - Line Segments
- Evaluate Trig functions of General Angles - CP
- Right Triangle, Non - Right Triangle & Unit Circle Trig - Topical Review - CP
- Law of Sines - Students Notes - CP
- Law of Cosines - Student Notes - CP
- Right Triangle, Non - Right Triangle & Unit Circle Trig - Exam Review
  - Right Triangle, Non - Right Triangle & Unit Circle Trig - Exam Review Answers - CP
- Right Triangle & Unit Circle Trig Exam Review - CP
  - Right Triangle & Unit Circle Trig Exam Review Answers - CP
- Linear and Angular Speed Word Problems
- Discovering Radian Measure Lab
- Right Triangle Trig - Student Notes - CP
- Reference Angles - CP
- Khan Academy - Right Triangle Trig
- Khan Academy - Non-Right Triangle Trig
- Khan Academy - Unit Circle

Verona High School

PreCalculus

Unit 6: Graphing Trigonometric Functions, Solving Trigonometric Equations, and Combined Sinusoids

Unit Duration: 4 Weeks

Stage 1: Desired Results

Established Goals:

F.IF.4 - For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities and sketch graphs showing key features given verbal descriptions of the relationship.

F.IF.7 - Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

F.IF.8 - Write a functions defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

F.IF.9 - Compare properties fo two functions each represented in a different way (algebraically, graphically, numerically in tables or by verbal descriptions).

F.BF.1 - Write a function that describes a relationship between two quantities.

F.BF.3 – Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

F.TF.5 - Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

F.TF.6 - Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

F.TF.7 - Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the contex.

F.TF.9 - Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems

A.REI.2 - Understand solving equations as a process of reasoning and explain the reasoning.

Transfer Goal:

Students will be able to <u>independently</u> use their learning to…

- apply their knowledge of the graphs of periodic functions and their equivalencies to recognize and analyze real-life periodic phenomena that have these properties.

Students will understand that:

U1. Changes to the algebraic equation of a function cause predictable changes to the function's graph
U2. Basic properties of trig graphs can be linked to a deep understanding of the unit circle and fundamental trig identities
U3. The symmetry of the unit circle allows trig equations to often have multiple answers
U4. Trig equations can be treated like much more basic equations by using the concept of substitution
U5. Trigonometric functions are the basis of many Physics properties including sounds waves and simple harmonic motion.
U6. Chords played by an instrument are the product of combined sine waves.
U7. The relationship between a function and an argument is not multiplication.
U8. Trigonometric Equations have infinitely many solutions
U9. The composition of ordinates is merely the sum or product of the ordinates of a function

Essential Questions:

Q1. What is the relationship between a trig equation and the trig function's graph?
Q2. How can the unit circle begin to explain the graphs of sin, cos, tan, cot, sec and csc?
Q3. How can we determine if an arc trig function will yield multiple angles in a given interval?
Q4. What is the algebraic and graphical relationship between trig equations and quadratic/linear equations?
Q5. What is the relationship between sounds waves, simple harmonic motion and trig functions?
Q6. Why do trigonometric equations have infinitely many solutions?
Q7. Why can't you distribute a function to corresponding pieces of its argument?
Q8. What is the algebraic and graphical interpretation of function composition?

Students will know:

K1. Period formulas
K2. Amplitude formulas
K3. Frequency formulas
K4. Equations of the midline
K5. Relationship between frequency, amplitude, volume and pitch for different trig functions
K6. 5 Critical Points
K7. Translations of Trig Functions
K8. Fundamental Trig Identities
K9. Inverse trig functions
K10. Sum and Difference Formulas
K11. Double and Half Angle Formulas

Students will be able to:

A1. Graph sin and cos waves
A2. Graph tan and cot waves
A3. Graph sec, csc waves
A4. Graph arctrig waves
A5. Graph waves by hand or on a calculator
A6. Verify Trigonometric Identities
A7. Solve systems of trig equations
A8. Solve linear trig equations
A9. Solve quadratic trig equations
A10. Solve trig equations with more than one function
A11. Solve trig equations graphically
A12. Solve multiple angle trig problems
A13. Derive trig formula
A14. Compose ordinates of sinusoidal graphs
A15. Analyze and write the equations for sinusoidal graphs

Stage 2: Acceptable Evidence

Performance Task & Unit Assessments

Students will show that they really understand by evidence of:

Assessments:

- Graphing Trig Functions - Quizzes
- Solving Trig Equations - Quizzes
- Verifying Trig Identities Quiz
- Combined Sinusoids Quiz
- Graphing Trig Equations Exam - CP
- Solving Trig Equations Exam - Student Answer Sheet
- Solving Trig Equations Exam - Version 1 - Honors
- Solving Trig Equations Exam - Version 2 - Honors
- Solving Trig Equations Exam - Version 3 - Honors
- Solving Trig Equations Exam - Version 4 - Honors
- Solving Trig Equations Exam - Version 5 - Honors
- Solving Trig Equations Exam - Version 6 - Honors
- Solving Trig Equations Exam - Version 7 - Honors
- Solving Trig Equations Exam - Version 1 - CP
- Solving Trig Equations Exam - Version 2 - CP
- Solving Trig Equations Exam - Version 3 - CP
- Combined Sinusoids Exam - Student Answer Sheet
- Combined Sinusoids Exam - Version 1 - Honors
- Combined Sinusoids Exam - Version 2 - Honors
- Combined Sinusoids Exam - Version 3 - Honors
- Combined Sinusoids Exam - Version 4 - Honors
- Combined Sinusoids Exam - Version 5 - Honors
- Combined Sinusoids Exam - Version 6 - Honors
- Combined Sinusoids Exam - Version 1 - CP
- Combined Sinusoids Exam - Version 2 - CP
- Combined Sinusoids Exam - Version 3 - CP

Transfer Task:

- Graphing Trig Functions Project - Students will analyze average monthly temperatures (or another sinusoidal set of data) in a city of their choice. They will apply their knowledge of periodic functions to analyze this periodic phenomenon
- Students will analyze dissonant and consonant sound intervals. Students will analyze intervals that sound pleasing and intervals that sound displeasing, and students will uncover the relationship between frequency and pitch, as well as amplitude and volume.

Other Evidence:
Students will show that they have achieved Stage 1 goals by:

Formal:
- Providing written/oral response to the EQs
- Passing all quizzes on basic concepts in unit.
-

Informal:
- Graphing Parent and Translated Trig Functions With(CP)/Without(HONORS) use of cheat sheet
- Solving all trig equations to generate multiple solutions (CP)/equations of solutions (HONORS)
- Recognizing Combined Sinusoids and their distinct properties
- Students should also troubleshoot examples and explain misapplications of the conventions or properties.

Reference Materials

- Pre-Calculus with Limits, a Graphical Approach
- TI-Nspire Lesson Activities: https://education.ti.com/en/timathnspired/us/precalculus

Graphing Trigonometric Functions
- Summary - Graphing Sine and Cosine
- Investigate Changes to Trig Graphs
- Graphing Trig Functions - Investigation Problem Set
- Graphing Trig Functions - Review
    - Graphing Trig Functions - Review Answers
- Graphs of Sound Waves for Last Section of Graphing Trig Functions
- Graphing Trig Functions - Teacher Notes
- Graphing Trig Functions - Student Notes
- Trig Graph Transformations Worksheet
    - Trig Graph Transformation Worksheet - Answers
- Graphing Trig Functions - Review - CP
- Steps for Graphing Trig Functions - CP
- Trig Identities - CP
- Khan Academy - Trig Graphs

Solving Trig Equations
- Solving Trig Equations Investigation
- Solving Trig Equations - Exam Review
    - Solving Trig Equations - Exam Review Answers
- Solving Trig Equations - Teacher Notes - Honors
- Solving Trig Equations - Student Notes - Honors
- Khan Academy - Trig Equations

Combined Sinusoids

- Combined Sinusoids Identity Practice
- Combined Sinusoids Problems
- Combined Sinusoids - Student Notes - Honors
- Combined Sinusoids - Teacher Notes - Honors
- Combined Sinusoids Formula Summary
- Combined Sinusoids Investigation
- Combined Sinusoids Review
- Combined Sinusoids - Multiple Angle Formulas

Verona High School — PreCalculus

Unit 7: Conic Sections, Parametric Equations, and Polar Coordinates	Unit Duration: 3 Weeks

Stage 1: Desired Results

Established Goals:

A.CED.3 - Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.

G.GPE.1 - Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

G.GPE.2 - Derive the equation of a parabola given a focus and directrix.

G.GPE.3 - Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant

Transfer Goal:

Students will be able to <u>independently</u> use their learning to…

- Model using parametric equations joining both Physics and Calculus topics.

Students will understand that:

U1. Parametric equations allow an independent time (or angle) variable to dictate both x and y position
U2. Polar coordinates are an alternative way to specify location on the plane
U3. Equations of conic sections can be reorganized and analyzed to produce adequate graphs
U4. Common shapes known in Geometry have complex Algebraic equations.
U5. Conic sections, parametric equations and the polar coordinate system are the basis of higher level Physics applications

Essential Questions:

Q1. What is the benefit of using parametric equations? Rectangular equations?
Q2. In what other disciplines are Pre-Calculus topics applicable?
Q3. What is the benefit of using polar coordinates? Rectangular coordinates?
Q4. What is the importance of conic sections?
Q5. Is a graph that overlaps itself possible, if it only has one equation?

Students will know:

K1. Standard form of a circle
K2. Standard form of a parabola
K3. Standard form of an ellipse
K4. Standard form of a hyperbola
K5. Formulas to convert a polar equation to rectangular
K6. Formulas to convert a rectangular equation to a polar equation
K7. Strategies to convert a rectangular equations to parametric
K8. Strategies to convert a parametric equation to a parametric

Students will be able to:

A1. Graph polar equations
A2. Convert from polar to rectangular
A3. Convert from rectangular to polar
A4. Graph parametric equations
A5. Eliminate the parameter
A6. Rewrite parametric or polar equations
A7. Identify Conics from graph OR equation in any form
A8. Write standard equations of conics
A9. Graph conics

Stage 2: Acceptable Evidence

Performance Task & Unit Assessments

Students will show that they really understand by evidence of:

Assessments:

- Parametrics Quiz
- Conics Quiz
- Conics Quizzes
- Conics, Parametrics and Polar Exam - Student Answer Sheet
- Conics, Parametrics and Polar Exam - Version 1 - Honors
- Conics, Parametrics and Polar Exam - Version 2 - Honors
- Conics, Parametrics and Polar Exam - Version 3 - Honors
- Conics, Parametrics and Polar Exam - Version 4 - Honors
- Conics, Parametrics and Polar Exam - Version 5 - Honors
- Conics, Parametrics and Polar Exam - Version 6 - Honors
- Conics, Parametrics and Polar Exam - Version 1 - CP
- Conics, Parametrics and Polar Exam - Version 2 - CP
- Conics, Parametrics and Polar Exam - Version 3 - CP

Transfer Task:

- Polar Coordinates - Students will create and analyze a polar design of their choice, focusing first on the artistic nature, then the mathematical properties.
- Parametric Equations - Students will use parametric equations to model the path of a projectile integrating concepts from Algebra 2 and Physics while introducing ideas that will be seen in Calculus.

Other Evidence:
Students will show that they have achieved Stage 1 goals by:

Formal:
- Providing written/oral response to the EQs
- Passing all quizzes on basic concepts in unit.
-

Informal:
- Converting between different coordinate systems With(CP)/Without(HONORS) use of cheat sheet
- Recognizing and Analyzing different conics by equations in any form
- Students should also troubleshoot examples and explain misapplications of the conventions or properties

Reference Materials

- Pre-Calculus with Limits, a Graphical Approach
- TI-Nspire Lesson Activities: https://education.ti.com/en/timathnspired/us/precalculus
- Folding Conic Sections Activity
- Conics Formula Sheet
- Conics Quiz Review
  - Conics Review - Answers
- Polar Worksheet
- Elementary Polar Worksheet

- Polar Graph Paper
- Polar Graph Paper
- Conics, Parametric and Polar HW Packet
  - 9.4 HW Packet Answers
  - 9.5 HW Packet Answers
  - 9.6 HW Packet Answers
- Parametrics Review Answers
- Conics, Parametric and Polar Exam Review
  - Conics, Parametric and Polar Exam Review Answers
  - Conics, Parametric and Polar Exam Review Answers
- Conics, Parametric and Polar - Student Notes - Honors
- Conics, Parametric and Polar - Teacher Notes - Honors
- Khan Academy - Conics
- Khan Academy - Parametric Equations

Verona High School — PreCalculus

Unit 8: Sequences & Series and Limits	Unit Duration: 4 Weeks

Stage 1: Desired Results

Established Goals:

A.SSE.4 - Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*

A.APR.5 - Know and apply the Binomial Theorem for the expansion of $(x+y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.

F.IF.3 - Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) +f(n-1) for n ≥1.*

F.BF.2 - Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

F.IF.7 - Analyze functions using different representations.

Transfer Goal:

Students will be able to <u>independently</u> use their learning to…

- Rationalize the behavior of patterned events to decide whether there will be an end to the pattern.

Students will understand that:

U1. Mathematical induction is a valid form of proof for sequence-type problems
U2. Sequence and series formulas are common-sense shortcuts
U3. Some infinite series converge
U4. A function does not have to be defined at a point for its limit to exist at that point
U5. Limits only exist if the two one-sided limits are equal

Essential Questions:

Q1. What is proof?
Q2. How could the sum of an infinite number of numbers possibly converge?
Q3. What is a limit?
Q4. How do we know if a limit exists?
Q5. What is the importance of the concept of continuity? The Intermediate Value Theorem?

Students will know:

K1. Formulas for nth terms of arithmetic and geometric sequences
K2. Formulas for finite arithmetic and geometric sequences
K3. Formulas for infinite geometric series
K4. Basics of Microsoft Excel
K5. Properties of Limits
K6. Definition of limit
K7. Intermediate Value Theorem

Students will be able to:

A1. Find nth terms of sequences
A2. Evaluate series
A3. Prove statements by mathematical induction
A4. Evaluate summations
A5. Set up spreadsheets in Excel
A6. Determine if a limit exists
A7. Find limits graphically and algebraically
A8. Determine intervals over which a function is continuous

Stage 2: Acceptable Evidence

Performance Task & Unit Assessments

Students will show that they really understand by evidence of:

Assessments:

- Sequences and Series Quiz - Honors
- Sequences and Series Exam - CP
- Sequences and Series Exam - Student Answer Sheet
- Sequences and Series Exam - Honors
- Limits Quiz - CP
- Limits Exam - CP
- Limits Exam - Version 1 - CP
- Limits Exam - Version 2 - CP
- Limits Exam - Version 3 - CP
- Limits Exam - Version 1 - Honors
- Limits Exam - Version 2 - Honors
- Limits Exam - Version 3 - Honors
- Limits Exam - Version 4 - Honors
- Limits Exam - Version 5 - Honors

Transfer Task:

Credit Card Project - Students will be presented with the fine print of a credit card special offer. They will determine the amount of money they would have to pay for an item if they a) only make minimum monthly payments, or b) miss a payment and go into default focusing on the topics of series and limits

Other Evidence:
Students will show that they have achieved Stage 1 goals by:

Formal:
- Providing written/oral response to the EQs
- Passing all quizzes on basic concepts in unit.
-

Informal:
- Recognizing arithmetic/geometric/sequences/series
- Analyzing limits in table, equation and graphical forms.
- Students should also troubleshoot examples and explain misapplications of the conventions or properties

Reference Materials

- Pre-Calculus with Limits, a Graphical Approach
- TI-Nspire Lesson Activities: https://education.ti.com/en/timathnspired/us/precalculus

Sequences and Series:

- Sequences and Series Project - CP
- Sequences and Series Formula Sheet - CP
- Geometric Sequences and Series Lab - CP

- Sequences and Series - Section 1 - Student Notes - CP
  - Sequences and Series - Section 1 HW Pages
- Sequences and Series - Section 2 - Student Notes - CP
  - Sequences and Series - Section 2 HW Pages
- Sequences and Series - Section 3 - Student Notes - CP
  - Sequences and Series - Section 3 HW Pages
- Sequences and Series - Section 4 - Student Notes - CP
  - Sequences and Series - Section 4 HW Pages
- Sequences and Series - Section 5 - Student Notes - CP
  - Sequences and Series - Section 5 HW Pages
- Sequences and Series Exam Review - CP
  - Sequences and Series Exam Review - Answers - CP
- Sequences and Series Practice - CP
- Sequences and Series Paper Tearing Activity
- Sequences and Series - Teacher Notes - Honors
- Sequences and Series - Student Notes - Honors
- Sequences and Series Exam Review - Honors
  - Sequences and Series Exam Review - Answers - Honors
- Sequences and Series Formula Sheet - Honors
- Khan Academy - Sequence & Series

Limits:

- Limits - Curve Sketching Activity
- Limits - Physics Activity
- Limits HW Packet
- Limits Exam Review - Honors
  - Limits Exam Review - Answers - Honors
- Limits Exam Review - CP
  - Limits Exam Review - Answers - CP
- Limits - Teacher Notes - Honors
- Limits - Student Notes - Honors
- Limits - Student Notes - CP
- Khan Academy - Limits